The Research on Enterprise Location——Based on Circle City Model

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Abstract: This paper get the inspiration in Hotelling's "linear city" location problem to explore the more realistic model "circle city", at the same time Weaken the exogenous conditions constraints such as price. Design the theoretical model of the enterprise site selection, in that case this paper modeling the factor of industrial agglomeration. Factors that affect the manufacturers' site and distance-related cost factors, as well as market-related cost factors. This paper has certain significance to the development of industrial zone.

Keywords: circle city, industrial agglomeration

I. Introduction

The formation of industrial zone is affect by both the historical factor and the accidental factors. Industrial zone is the result of the combined effect. Agglomeration effects of factors will contribute to the formation and transfer of industrial zone. Agglomeration factor is an important factor in the transfer of industrial zone. Without the selection of the enterprise site, it is hard to research on the industrial zone. So, this paper will angle down to the enterprise site selection of this micro-level research.

Hotelling (1929) first set up a location selection model to show the spatial characteristics of different products appeal to consumers, start from the description of the spatial location of different enterprises. In a length of one of the "linear city" model, there are two enterprises or store selling homogeneous products. Assuming uniform distribution of consumers considering only the transportation cost minimization, in the situation that the price is given exogenous, the two enterprises select the location in a length of one of the "linear city" model. Because the price and marginal price is fixed, therefore the goal of the enterprise selection is to maximize the demand. we set Enterprise 1 at a point, Enterprise 2 locate at $1-b$ point, suppose Enterprise 1 always locate at the left side of the Enterprise 2,namely $a \le 1-b$ .Apparently the Enterprise 1 has an motivation moving to the right side to enlarge the sales volume. Enterprise 1's Sales volume at this time is $a+(1-b-a)/2$, it will increase with the a increase, but $1-b$ is the upper limit of a .Enterprise 2 also has the motivation to moving to the left side, the only equilibrium will get when $a=b=1/2$. Hotelling's "linear city" location problem is based on no price competition. The dynamic competition results of profit maximization results are: the 2 enterprise tend to gather in the center of the city. In this case, the selection of location has known as the Nash equilibrium strategy [1].

Hotelling's "linear city" model has many variations. One of the many variations is Bertrand - Nash equilibrium based on endogenous prices. The two stage of game is: select location first, select price secondly. When the transportation cost is linear form, the model has no solution. If the cost is quadratic form, then there exists a two-stage game of separating equilibrium, each enterprise will choose the location away from the opponent (located at both ends of the linear city).Another one of the variation is Cournot - Nash equilibrium based on endogenous productivity. The two-stage game is: select location first, select productivity secondly. In this model, if the distribution function of the overall product cost is convex, Agglomeration equilibrium exists in the region of the minimum production cost and the minimum transport cost point. Hotelling's model has been ignored for a long time until the 20th century 70s. With the rise of game theory, people began to realize that hidden in the Cournot, Bertrand and Hotelling model of the extraordinary power of game theory [2].

Beckmann and Stem (1972) used a very precise method to formal solve the substitution relationship between the transportation cost and increasing compensation under conditions of space competition. Salop (1979) made an exquisite circle city model, consumers uniformly distributed in the circumference of a ring boundary, along the circumference of the distribution of enterprises. The two stage of game is: first stage, Potential entrants choose whether to enter the market. If the number N enterprises choose to enter but do not select location, they automatically locate in the circle equidistant, then the maximizing difference generated exogenously. Second stage, after the condition that they choose to enter, the enterprises choose to have price competition. Compared to cities with the linearization, this model is more suited to the situation with no extreme features, while the linear city model will help analyze the distribution characteristics are extreme circumstances. Circle city model has been improved by many scholars then. When analyzing the two stage circle city model, Pal (1998) found that the conclusions are totally opposite to the result obtained with the linear city model. He pointed out that the two companies either Bertrand competition or Cournot competition, the two companies will be distributed in a circle equidistant from the two ends of the diagonal, that is gathering does not occur. Matsushima (2001) found that, if there is n such a two-stage companies to compete, there will be half of the enterprise gather at the circle at one end point of the diagonal, the other half business gathering in another endpoint[3].

In this paper, I want to take a further study of Salop circle city model , proposed further study of such factors as the price endogenous from exogenous change, the introduction of product cost distribution functions, to further explore the circle city model of the location and yield of two-stage selection model .

II. Basic model construction

Two-stage location selection and productivity choose Cournot competition model construction: Assume that countless number of consumers evenly distributed in the circle ring city with circumference of 1. Enterprises are selling their products on the circle ring city. The first stage of game, Enterprise 1 and Enterprise 2 choose the location x1 and x2 on the circle ring city. The second stage of game, Enterprise provides different output to compete. The precondition is that enterprise will provide different price at different point on the circle ring city [4].

The circle starting point 0 and end point 1 coincidence, the location x presents the length along the circle ring from point 0 in anticlockwise. Assume demand function is linear at arbitrary point x, then Q(x)=[a-P(x)]/b, so its price function is P(x)=a-b*Q(x),and a>0,b>0 always constant, Q(x) is the total output that enterprise provide for Point x, if qi(x,x1,x2),(i=1,2) present the number of i enterprise provide output at point x, then $Q(x)=q_1(x,x_1,x_2)+q_2(x,x_1,x_2)$.Enterprises in which the x_i position to produce and transport products to point x, would like to stress is the product transportation along the circle close to transport, that is to choose the distance should be a circle of minor arc. When the enterprise at x_i is providing service to *x,* transportation cost is proportional to transport distance, With *k* as units of product transportation cost, $|x-x_i|$ presents the distance between enterprise *i* and sales point *x*, when $|x-x_i|<1/2$,enterprise *i* linear transportation cost is $k|x-x_i|$, when $|x-x_i|\geq 1/2$,enterprise *i* linear transportation cost is $k|1-x+x_i|$,*i*=1,2. The endogenous cost of enterprise *i* producing the product at point x_i is $c(x_i)$,*i*=1,2. Suppose two enterprises on the location and production of select two means to make optimal strategy to achieve the business with their respective profit-maximizing principles. From the condition above ,we can conclude the price function at x is $P(x)=a-b[q_1(x, x_1, x_2)+q_2(x, x_1, x_2)]$ .In order to facilitate the future use, we use $q_i(x)$ instead of $q_i(x,x_1,x_2)$ [5].

III. Cournot model

Cournot model is based on the assumption that companies first conduct site selection, and then to output a variable to make optimal strategy[6]. The optimal strategy of enterprises not only depends on the location, but the choice also depends on the location of production location at the unit cost of product distribution function. Compare to the location of each enterprise, the output strategy is independent. For each point x, the Cournot equilibrium point represent by a group of independent Cournot equilibrium point. Set $\pi_i(x,x_1,x_2)$ presents the profit that enterprise *i* get at point x based on two companies competition, *i*=1,2. In that case ,the profit that they get at point x is

$$\pi_i(x, x_1, x_2)=[a-bQ(x)-k\,|\,x-x_i\,|-c(x_i)]q_i(x), \quad i=1,2 \text{ and } |x-x_i|<1/2 \tag{1}$$

$$\pi_i(x, x_1, x_2)=[a-bQ(x)-k\,|\,1-x+x_i\,|-c(x_i)]q_i(x), \quad i=1,2 \text{ and } |x-x_i|\geq 1/2 \tag{2}$$

$[a-bQ(x)-k\,|\,x-x_i\,|]q_i(x)$ or $[a-bQ(x)-k\,|\,1-x+x_i\,|]q_i(x)$ presents the profit the enterprises get at the point x when regardless of the cost, and $c(x_i)q_i(x)$ present enterprise *i* total produce cost at point x_i.Two companies to pursue profit maximization objectives is:

$$\max_{q_i(x)}\pi_i(x;x_1,x_2)=[a-bQ(x)-k\,|\,x-x_i\,|-c(x_i)]q_i(x) \quad i=1,2 \text{ and } |x-x_i|<1/2 \tag{3}$$

$$\max_{q_i(x)}\pi_i(x;x_1,x_2)=[a-bQ(x)-k\,|\,1-x+x_i\,|-c(x_i)]q_i(x) \quad i=1,2 \text{ and } |x-x_i|\geq 1/2 \tag{4}$$

(3) and (4) Seeking partial derivative to $q_i(x)$,Make it zero in order to be the optimal strategy for both companies the need for a first-order condition is:

$$a-bQ(x)-bq_i(x)-k\,|\,x-x_i\,|-c(x_i)=0\,, \quad i=1,2 \text{ and } |x-x_i|<1/2 \tag{5}$$

$$a-bQ(x)-bq_i(x)-k\,|\,1-x+x_i\,|-c(x_i)=0 \quad i=1,2 \text{ and } |x-x_i|\geq 1/2 \tag{6}$$

Subdivided (5) into two equations:

$$a-b[q_1(x)+q_2(x)]-bq_1(x)-k\,|\,x-x_1\,|-c(x_1)=0 \quad i=1,2 \text{ and } |x-x_i|<1/2 \tag{7}$$

$$a-b[q_1(x)+q_2(x)]-bq_2(x)-k\,|\,x-x_2\,|-c(x_2)=0 \quad i=1,2 \text{ and } |x-x_i|<1/2 \tag{8}$$

Collate (7) and (8),we can conclude the enterprise *i* output at point x and the total output of the two enterprise under the condition $|x-x_i|<1/2$ is:

$$q_i(x)=\frac{a+k(|x-x_1|+|x-x_2|)-3k|x-x_i|+c(x_1)+c(x_2)-3c(x_i)}{3b} \tag{9}$$

$$Q(x)=\frac{2a-k(|x-x_1|+|x-x_2|)-[c(x_1)+c(x_2)]}{3b}, \tag{10}$$

When $|x-x_i|\geq 1/2$,we use the same method to calculate the enterprise *i* output at point x and the total output of two enterprise produce at the point x is:

$$q_i(x)=\frac{a+k(|1-x+x_1|+|1-x+x_2|)-3k|1-x+x_i|+c(x_1)+c(x_2)-3c(x_i)}{3b} \tag{11}$$

$$Q(x)=\frac{2a-k(|1-x+x_1|+|1-x+x_2|)-[c(x_1)+c(x_2)]}{3b} \tag{12}$$

Put (9) and (10) into (1), put (11) and (12) into (2):

$$\pi_i(x,\ x_1,\ x_2)=\frac{[a-k(|x-x_1|+|x-x_2|-3k|x-x_i|+c(x_1)+c(x_2)-3c(x_i))]^2}{9b}$$

$|x\text{-}x_i|<1/2$ (13)

$$\pi_i(x,\ x_1,\ x_2)=\frac{[a-k(|1-x+x_1|+|1-x+x_2|-3k|1-x+x_i|+c(x_1)+c(x_2)-3c(x_i))]^2}{9b}$$

$|x\text{-}x_i|\geq 1/2$ (14)

In Cournot model, the two enterprise total profit at the circle is:

$$\prod_i(x_1,\ x_2)=\int_0^1\pi_i(x,\ x_1,\ x_2)dx,\qquad i=1,2 \qquad (15)$$

Here we can see that the enterprise 2 is equivalent to 1, so we only need to discuss one enterprise. Then we need to calculate the maximum of (15), the location of an enterprise may be two situations, one is the location of the two companies fall on the same half of the circle, the other possibility is that two different companies fall within the semi-circle. First we discuss the first situation, x_1 and x_2 fall on same semi-circle.

We assume $x_1\leq x_2$,divide a circle into five regions$[0,x_1]$,$[x_1,x_2]$,$[x_2,x_1+1/2]$,$[x_1+1/2,x_2+1/2]$,$[x_2+1/2,1]$, (15) can expressed as :

$$\begin{aligned}\prod_1(x_1,x_2)=\frac{1}{9b}\Big\{&\int_0^{x_1}[a-2k(x_1-x)+k(x_2-x)-2c(x_1)+c(x_2)]^2dx\\&+\int_{x_1}^{x_2}[a-2k(x-x_1)+k(x_2-x)-2c(x_1)+c(x_2)]^2dx\\&+\int_{x_2}^{x_1+\frac{1}{2}}[a-2k(x-x_1)+k(x-x_2)-2c(x_1)+c(x_2)]^2dx\\&+\int_{x_1+\frac{1}{2}}^{x_2+\frac{1}{2}}[a-2k(1-x+x_1)+k(x-x_2)-2c(x_1)+c(x_2)]^2dx\\&+\int_{x_2+\frac{1}{2}}^{1}[a-2k(1-x+x_1)+k(1-x+x_2)-2c(x_1)+c(x_2)]^2dx\Big\}\end{aligned}$$

As the calculation of volume is too large, we use Mathematica5.0 software to calculate the definite integral of each the following results:

$$\int_0^{x_1}[a-2k(x_1-x)+k(x_2-x)-2c(x_1)+c(x_2)]^2dx=\frac{-[a-2c(x_1)+c(x_2)-2kx_1-kx_2]^3+[a-2c(x_1)+c(x_2)-kx_1-kx_2]^3}{3k}$$

$$\int_{x_1}^{x_2}[a-2k(x-x_1)+k(x_2-x)-2c(x_1)+c(x_2)]^2dx=\frac{-[a-2c(x_1)+c(x_2)+2kx_1-2kx_2]^3+[a-2c(x_1)+c(x_2)-kx_1+kx_2]^3}{9k}$$

$$\int_{x_2}^{x_1+\frac{1}{2}}[a-2k(x-x_1)+k(x-x_2)-2c(x_1)+c(x_2)]^2dx=\frac{[a-2c(x_1)+c(x_2)+2kx_1-2kx_2]^3-[a-2c(x_1)+c(x_2)-k/2+kx_1-kx_2]^3}{3k}$$

$$\begin{aligned}&\int_{x_1+\frac{1}{2}}^{x_2+\frac{1}{2}}[a-2k(1-x+x_1)+k(x-x_2)-2c(x_1)+c(x_2)]^2dx\\&=\frac{1}{72k}\{-2[a-2c(x_1)+c(x_2)+(-1+2x_1-2x_2)k]^3\\&+[2a-4c(x_1)+2c(x_2)+(-1-4x_1+4x_2)k]^3\}\end{aligned}$$

$$\begin{aligned}&\int_{x_2+\frac{1}{2}}^{1}[a-2k(1-x+x_1)+k(1-x+x_2)-2c(x_1)+c(x_2)]^2dx\\&=-\frac{1}{3k}\{[a-2c(x_1)+c(x_2)-2kx_1+kx_2]^3\\&-1/8[2a-4c(x_1)+2c(x_2)-k-4kx_1+4kx_2]^3\end{aligned}$$

The sum of all five integral got a total profit is:

$$\begin{aligned}\prod_1(x_1,x_2)=\frac{1}{108b}\{&96k^2x_1^2x_2^2+24k^2x_1^2-48k^2x_1x_2+24k^2x_2^2-96k^2x_1^2x_2\\&+k^2-32k^2x_2^3+32k^2x_1^3-6k[a-2c(x_1)+c(x_2)]+12[a-2c(x_1)+c(x_2)]^2\}\end{aligned}\qquad(15)$$

The purpose of Enterprise 1 located at x_1 is maximum the profit, which is maximum $\prod_1(x_1,x_2)$.

To calculate $\max\limits_{0\leq x_1<1}\prod(x_1,x_2)$, we need calculate through seeking partial derivative of x_1.

$$\begin{aligned}\frac{\partial\prod_1(x_1,x_2)}{\partial x_1}&=\frac{4k^2}{9b}(x_2-x_1)(-1-2x_1+2x_2)\\&+\frac{1}{9b}[k-4a+8c(x_1)-4c(x_2)]c'(x_1)\end{aligned}\qquad(16)$$

To determine whether there is a maximum, we seeking the second-order partial derivatives of $\prod(x_1,x_2)$ on x_1:

$$\begin{aligned}\frac{\partial^2\prod_1(x_1,x_2)}{\partial x_1^2}&=\frac{16k^2}{9b}[(x_1-x_2)+\frac{1}{4}]\\&+\frac{1}{9b}[k-4a+8c(x_1)-4c(x_2)]c''(x_1)+\frac{8}{9b}[c'(x_1)]^2\end{aligned}\qquad(17)$$

Another case, if $x_1=x_2$,means two enterprise located at the same point, through make a first-order partial derivatives $\frac{\partial\prod_1(x_1,x_2)}{\partial x_1}=0$,we can calculate through (16)：

$$[k-4a+4c(x_1)]c'(x_1)=0\qquad(18)$$

If $k\text{-}4a+4c(x_1)\neq 0$,then $c'(x_1)=0$. Put it into (17)

$$\left.\frac{\partial^2\prod_1(x_1,x_2)}{\partial x_1^2}\right|_{x_1=x_2}=\frac{4k^2}{9b}+\frac{1}{9b}[k-4a+4c(x_1)]c''(x_1)\qquad(19)$$

And $\frac{\partial^2\prod_1(x_1,x_2)}{\partial x_1^2}<0$, In general, we believe that the total unit cost and freight price can't be greater than aggregate demand and price. Then $P(x)+bQ(x)>k+c>k/4+c$, so $P(x)+bQ(x)=a$, $a>k/4+c$,and $k\text{-}4a+4c(x_1)<0$,only if $c''(x_1)>\frac{4k^2}{4a-4c(x_1)-k}$,we can get $\frac{\partial^2\prod_1(x_1,x_2)}{\partial x_1^2}<0$.From $c'(x_1)=0$ and $c''(x_1)>\frac{4k^2}{4a-4c(x_1)-k}>0$

we can conclude：The cost function of enterprise on the circle is convex, and x_1 is the cost function of the minimum point of the corresponding independent variables. Of course, such a conclusion also set up an enterprise 2.

From the above conclusion can be that, when the unit of transportation costs k is very small. In the circle city, enterprise gathered at the minimum point of the cost distribution function. At the same time the two companies have reached their own profit maximization, without any agreement the two companies will be able to achieve the Nash equilibrium.

Another case is $x_2 = \frac{1}{2} + x_1$, the two enterprises are equidistant distribution, each located at one side of the circle. from (16) we can calculate：

$$[k - 4a + 8c(x_1) - 4c(x_2)]c'(x_1) = 0 \tag{20}$$

If k-4a+8c(x_1)-4c(x_2)≠0,then $c'(x_1)$=0. Put it into (17),we can calculate：

$$\left.\frac{\partial^2 \prod_1(x_1,x_2)}{\partial x_1^2}\right|_{x_1 = x_2 - \frac{1}{2}} = -\frac{4k^2}{9b} + \frac{1}{9b}[k - 4a + 8c(x_1) - 4c(x_2)]c''(x_1) \tag{21}$$

As same as the situation above, the cost function and the freight cost on circle should be less than the price and total demand. Assume k-4a+8c(x_1)-4c(x_2)<0 has a certain practical significance, so if $c''(x_1) >$ 0,we can conclude $\frac{\partial^2 \prod_1(x_1,x_2)}{\partial x_1^2} < 0$, $\prod_1(x_1,x_2)$ get profit maximum at x_1.

From c' (x_1)=0 and $c''(x_1) > 0$,we can conclude the enterprise 1 obtain the minimum value of distribution function of product cost at x_1.

If $k - 4a + 8c(x_1) - 4c(x_2) > 0$, when $c''(x_1)<0$, $\frac{\partial^2 \prod_1(x_1,x_2)}{\partial x_1^2} < 0$, $\prod_1(x_1,x_2)$ obtain the maximum profit at x_1. At this point , enterprise 1 obtain the maximum value of distribution function of product cost at the x1. that is, this point is not a Nash equilibrium point.

We can therefore conclude: In the cost distribution function for the strictly convex function of circumstances, when the transportation costs coefficient is relatively small（k-4a+8c(x_1)-4c(x_2)<0）.Enterprises will be concentrate of the minimum point of the cost distribution function and attain their profit-maximizing. If the enterprises are in a circle diameter of the symmetry point, and this time the cost of a function to obtain maximum value, respectively, this site is not a Nash equilibrium point. In other words, the cost of distribution function for the strictly convex function of circumstances, as long as transportation costs coefficient k is not particularly large, gathering will occur at the cost of the minimum point of the region and to attain their profit-maximizing. With the cost of transport coefficient k increase, driven by the pursuit of profit-maximizing firms may be breaking the status of such spatial concentration.

The biggest difference between this paper and others of the circle with such like Salop circular city model is the introduction of the cost of distribution function, rather than external to the fixed cost as a consideration. As the product cost, including production costs, transportation costs and transaction costs, while the production costs also include fixed costs, marginal costs and market entry costs, not only by the spatial concentration of corporate location underlying factors, but also by the spatial concentration of the market factors constraints.

IV. Conclusion

This paper has established a distribution function based on the product cost and yield of the two-stage site selection model for the two to compete. When an enterprise is located at the minimum cost of distribution function in which the location of the point, the profit-maximizing output decision-making will generate spatial agglomeration. When an enterprise is located equidistant from the circle diameter, only the minimum in the cost distribution function in which the point location, in order to achieve the profit-maximizing output decision-making. If the enterprise is located in the maximum of the cost of distribution function in which the location of the point, it will cause instability in production decisions at this time reach the Nash equilibrium locations.

References

[1] d'Aspremont,C.,J.Gabszewicz,and J.F.Thisse. 1979. On Hotelling's Stability in Competition [J]. Econometrica.1979(17)：1145-1151.

[2] Hamilton,J.H., Thisse,J.F.,Weskamp， A.,Spatial discrimination：Bertrand versus Cournot in a model of location choice[J]. Reginal Science and Urban Economics.1989(9)：87-102.

[3] Rauch.J."Does history matter only when it matters a little？The case of city-industry location,".Quarterly Journal of Economics 108.843-867 (1993).

[4] T.J.Holmes."How Industries Migrate When Agglomeration Economies are Important,"Journal of Urban Economics 45.240–263 (1999).

[5] Qi Liang. Industrial Cluster Theory. Commercial Press.2004.

[6] LiangCai He. Mathematical Economics and Management. Chongqing University Press.2006.